Calc I - Final Review

Exam I

1. We'd like to estimate
$$\lim_{t\to 0}\frac{5^t-1}{t}$$
so we generate the following table:

t	0.1	0.01	0.001	0.0001	0.00001
$f(t)$	1.746189	1.622459	1.610734	1.609567	1.609451

Write down the value of the limit to as many decimal places that you are confident of.

4. Find the derivatives of the following functions, *using the definition of the derivative.*
   (a) $f(x) = 3x^2 - 2x + 1$
   (b) $f(x) = \sqrt{x}$
5. Find the derivatives of the following functions, using the power rule, sum rule, and/or constant multiple rule.
   (a) $f(x) = 3x^2 - 2x + 1$
   (b) $f(x) = \sqrt{x}$
   (c) $f(x) = x^2(3x^2 - 2x + 1)$
   (d) $f(x) = (2x-3)^2$
6. Let $f(x) = x^4 - x^2$.
   (a) Find the derivative of f.
   (b) Write down an equation for the tangent line at $x = 2$.
7. Write down a complete sentence referring to the intermediate value theorem showing that the function $f(x) = x^5 + x^3 + x - 1$ has a root in the interval $(0, 1)$.
8. If f and g are differentiable functions, use the definition of the derivative to prove that
$$\frac{d}{dx}(f(x) + 2g(x)) = f'(x) + 2g'(x)$$

Exam II

1. Find the derivatives of the following functions.

(a) $f(x) = x^2 \ln(3x^4)$

(b) $f(x) = \sin(3x)\arcsin(x^3)$

2. Use the definition of the derivative to show that

$$\frac{d}{dx}x^2 f(x) = 2xf(x) + x^2 f'(x).$$

5. Use logarithmic differentiation to compute the derivative of $f(x) = x^x$.

7. Consider the equation $x^4 - 3xy + 2y^3 = 0$. Find an equation of the line that is tangent to the graph of this curve at the point $(1, 1)$.

8. Let $f(x) = 2x^3 - 3x^2 - 12x + 2$. Sketch the graph of f using the first derivative to identity the exact locations of any relative extremes and the second derivative to identity the exact locations of any inflection points.

Exam III

1. Use a linear approximation to find a good estimate to $\sqrt[4]{15.5}$.

3. Suppose I pull the bottom of a 12 foot tall ladder away from a wall at the rate of 3 feet per second. At what rate is the top of the ladder moving towards the floor when it is 2 feet away from the floor?

4. Suppose I set up a rectangular corral to enclose 4000 square feet with three inner partitions breaking the corral into four pieces, as shown in figure 1. The material for the exterior portion costs twice as much as the material for the interior walls. What are the dimensions of the cheapest such corral?

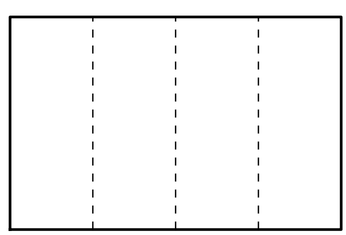

Figure 1: A partitioned corral

6. Evaluate the following indefinite integrals.

(a) $\displaystyle\int \big(x^4 - 8x^3 - \sin(x)\big)\, dx.$

(b) $\displaystyle\int \frac{(x-1)}{x^2} dx.$

7. Use the Fundamental Theorem of Calculus to evaluate

$$\int_0^3 \left(4x^3 - 2x\right) dx.$$

8. The complete graph of a function is shown in figure 2; it consists of two line segments and a semi-circle. Evaluate

$$\int_{-3}^3 f(x)\, dx.$$

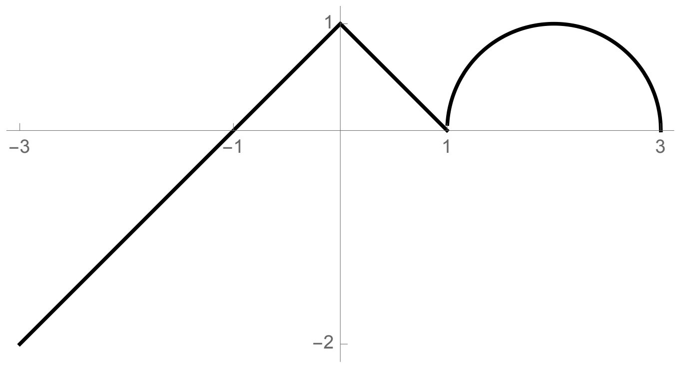

Figure 2: The complete graph of a function

u-Substitution

1. Use u-substitution to evaluate the following indefinite integrals.
   (a) $\int x\sqrt{x^2+1}\, dx$
   (b) $\int \sqrt{2x+1}\, dx$
   (c) $\int \sin^3(x)\cos(x)\, dx$
   (d) $\int \frac{1}{x\ln(x)}\, dx$
2. Use u-substitution to evaluate the following definite integrals.
   (a) $\int_1^2 x^2(x^3+1)^9\, dx$
   (b) $\int_{-1}^1 xe^{\sin(x^2)}\, dx$
3. Use u-substitution to express the following normal integral as a *standard* normal integral:

$$\frac{1}{\sqrt{2\pi}4}\int_{-1}^2 e^{-(x-1)^2/32}\, dx.$$