

# Calc I - Review for exam I

The first exam will be next Tuesday, September 17. Here are some problems that might help.

1. We'd like to estimate

$$\lim_{t \rightarrow 0} \frac{7^t - 1}{t}$$

so we generate the following table:

$t$	0.1	0.01	0.001	0.0001	0.00001
$f(t)$	2.14814	1.96497	1.94780	1.94610	1.94593

Write down the value of the limit to as many decimal places that you are confident of.

2. Let  $f(x) = x^2 + 2x - 1$ .
- Sketch the graph of  $f$ , together with the line tangent to the graph at the point  $(0, f(0))$ .
  - Compute the slope of the secant line through the points  $(0, f(0))$  and  $(2, f(2))$ . Draw this line on your graph from part (a).
  - Write down the difference quotient for  $f$  at 0, i.e. the slope of the secant line through the points  $(0, f(0))$  and  $(0 + h, f(0 + h))$ .
  - Compute the limit as  $h \rightarrow 0$  of your answer to part (c).
  - How does your answer to part (d) relate to your sketch from part (a)?
3. Let

$$f(x) = \frac{(5x - 1)(x - 3)}{x^2 - 9}.$$

Evaluate each of the following the limits.

- $\lim_{x \rightarrow -1} f(x)$
  - $\lim_{x \rightarrow -3^-} f(x)$ ,  $\lim_{x \rightarrow -3^+} f(x)$ , and  $\lim_{x \rightarrow -3} f(x)$
  - $\lim_{x \rightarrow 3} f(x)$
4. Figure 1 shows the complete graph of a function  $f$ ; its domain is  $(-1, 2]$ .
- What are  $f(1.5)$  and  $\lim_{x \rightarrow 1.5} f(x)$ ?
  - What are  $\lim_{x \rightarrow 1^-} f(x)$ ,  $\lim_{x \rightarrow 1^+} f(x)$ , and  $\lim_{x \rightarrow 1} f(x)$ ?
  - What can you say about  $\lim_{x \rightarrow -1^+} f(x)$ .
5. Find the derivatives of the following functions, *using the definition of the derivative*.
- $f(x) = 2x^2 - 4x$
  - $f(x) = 1/\sqrt{x}$
  - $f(x) = x^5$

6. Find the derivatives of the following functions, using the power rule, sum rule, and/or constant multiple rule.

(a)  $f(x) = 2x^2 - 4x$

(b)  $f(x) = 1/\sqrt{x}$

(c)  $f(x) = x^5$

(d)  $f(x) = x^5(x^2 - x - 1)$

(e)  $f(x) = (x^2 - x - 1)/x$

(f)  $f(x) = (x - 5)^2$

(g)  $f(x) = (x + 2)^3$

7. Let  $f(x) = x^2 - x$ .

(a) Find the derivative of  $f$ .

(b) Write down an equation for the tangent line at  $x = 2$ .

8. If  $f$  and  $g$  are differentiable functions, prove that

$$\frac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$$

9. Write down a complete sentence referring to the intermediate value theorem showing that the function  $f(x) = x^7 + x^3 + x + 1$  has a root in the interval  $(-1, 1)$ .

10. The complete graph of a function  $f$  is shown in figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of  $f'$ .

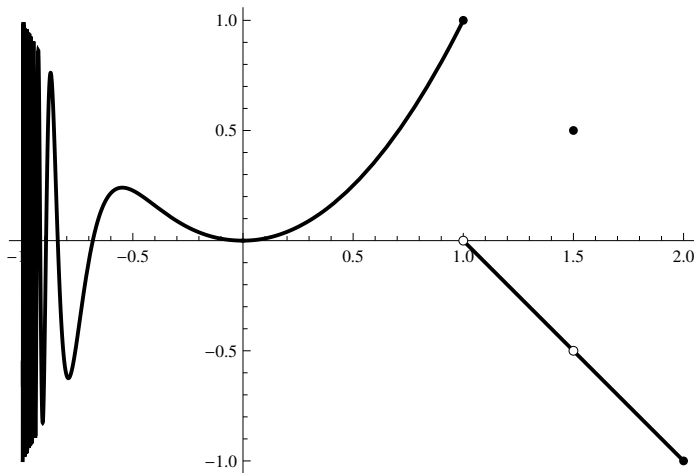


Figure 1: The graph for problem 6

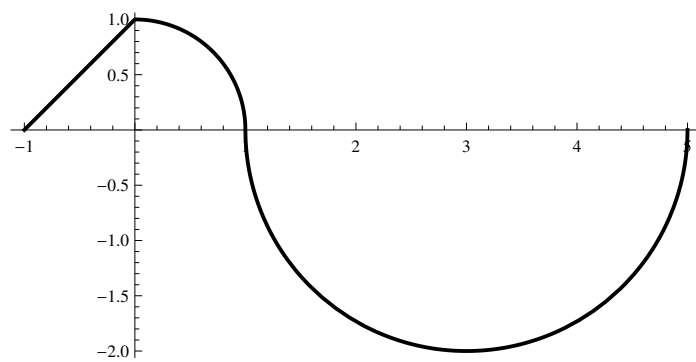


Figure 2: The graph for problem 11