Calc I - Review for exam II

The first exam will be next Tuesday, October 22. Here are some problems that might help.

1. Find the derivatives of the following functions.
   (a) $f(x) = \tan(x) + \sec(x) + \ln(x) + \arcsin(x) + \arctan(x)$
   (b) $f(x) = x\ln(3x)$
   (c) $f(x) = \tan(2x)\arctan(x^2)$
2. Use the definition of the derivative to show that
$$\frac{d}{dx}x^3 f(x) = 3x^2 f(x) + x^3 f'(x).$$
3. In this problem, we'd like to prove that
$$\frac{d}{dx}\cos(x) = -\sin(x),$$
using the definition of the derivative.
   (a) Write down the difference quotient for the cosine.
   (b) Use a trig identity to expand the $\cos(x+h)$ in your difference quotient.
   (c) Rearrange your expanded difference quotient so that you can take the necessary limits.

   Note that you may assume the following facts:
   - $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.
   - $\lim_{\theta\to 0}\sin(\theta)/\theta = 1$
   - $\lim_{\theta\to 0}(\cos(\theta)-1)/\theta = 0$
4. Use the formula for the derivatives of the sine and cosine together with the combination rules for differentiation to establish the following formulae.
   (a) $\displaystyle\frac{d}{dx}\tan(x) = \sec^2(x)$
   (b) $\displaystyle\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$.
5. Use the fact that $\lim_{\theta\to 0}\sin(\theta)/\theta = 1$ to compute the following limits.
   (a) $\displaystyle\lim_{x\to 0}\frac{\sin(3x)}{x}$
   (b) $\displaystyle\lim_{x\to 0}\frac{x}{\tan(2x)}$
   (c) $\displaystyle\lim_{x\to 0}\frac{1-\cos(x)}{x^2}$

6. Use logarithmic differentiation to compute the following derivatives.
   (a) $\frac{d}{dx}3^x$
   (b) $\frac{d}{dx}x^x$
   (c) $\frac{d}{dx}\sin(x)^{\cos(x)}$
7. Compute the derivative of the arccosine using the following recipe.
   (a) Write $y = \arccos(x)$, apply the cosine to both sides, and simplify.
   (b) Apply implicit differentiation to your response to part (a) and solve for y' in terms of y.
   (c) Express your solution to the previous part in term of x.
8. Consider the equation $x^3 + 2xy + y^4 = 4$. Find an equation of the line that is tangent to the graph of this curve at the point $(1, 1)$.
9. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 12x + 1$ on $[0, 3]$.
10. Let $f(x) = x^3 + 3x^2 - 24x - 1$.
    (a) Use the first derivative to find maximal intervals on which f is increasing or descreasing.
    (b) Use the second derivative to find maximal intervals on which f is concave up or concave down.
    (c) Use the information from the previous two parts to sketch a graph of f.
11. Let $f(x) = xe^{-x^2}$. A graph of f is shown in figure 1.
    (a) Find the exact locations of the absolute maximum and minimum of f.
    (b) Find the exact locations of the inflection points of f,

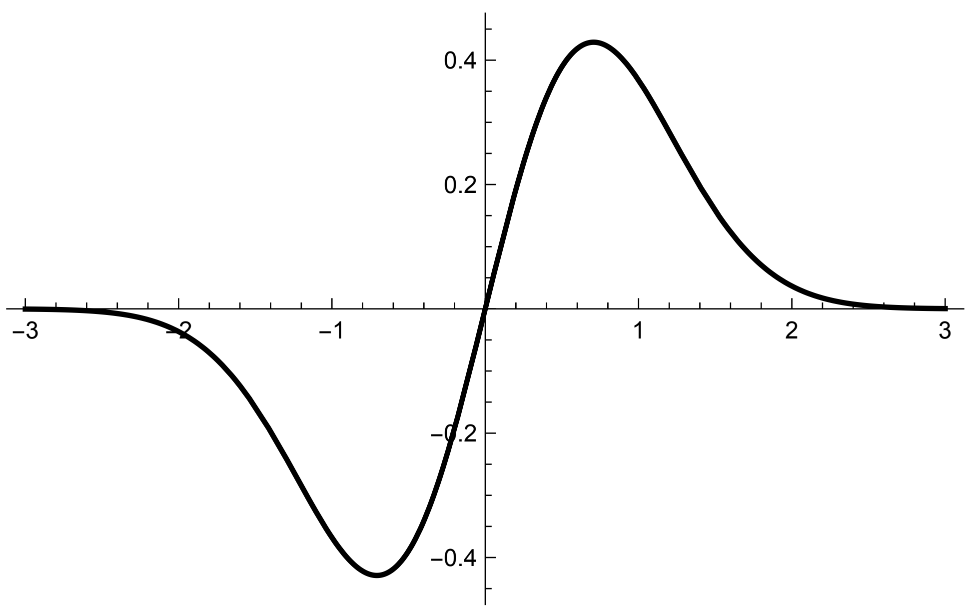

Figure 1: The graph of $f(x) = xe^{-x^2}$